Opening and Closing Operators in Fuzzy Morphology Using Conjunctive Uninorms

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Abstract

In this paper, a fuzzy mathematical morphology is defined using conjunctive uninorms. Implementation results for two special classes of representable and idempotent conjunctive uninorms are presented, proving that these classes become specially appropriate for edge detection. Moreover, open and closed fuzzy objects are defined using the mentioned kinds of uninorms which leads us to prove, for the case of representable ones, the generalized idempotence for fuzzy opening and closing.

Keywords: Fuzzy morphology, erosion, dilation, opening, closing, uninorms, implicators.

AIM

- Development of an approach to fuzzy mathematical morphology based on conjunctive uninorms.
- Study of the most suitable conjunctive uninorms to be used in this framework.
- Restriction: the most usual algebraic and morphological properties (from the classical approach) must be preserved.
- Tool: The general framework for fuzzy mathematical morphology constructed by De Baets [1997].

MOTIVATIONS

- The shapes in an image are not always crisply defined.
- Uncertainty can arise within each level of image analysis and pattern recognition.
- The method used in feature extraction should have a provision for representing and manipulating the uncertainties.
- Fuzzy set theory provides a mechanism to represent and manipulate uncertainty and ambiguity.
- Fuzzy operators and their properties have found considerable applications in image analysis and pattern recognition.

FUZZY LOGICAL OPERATORS

Three known classes of conjunctive uninorms:

- Uninorms in the class $\mathcal{U}_{\min}$
- Representable uninorms.
- Idempotent uninorms.

Which class of uninorms is the most suitable?

left-continuity is essential

$$\Downarrow$$

$$\begin{cases} \text{Representable uninorms,} \\ \text{Idempotent uninorms.} \end{cases}$$

THEN: WE WILL ONLY USE HERE REPRESENTABLE AND IDEMPOTENT, CONJUNCTIVE UNINORMS

FUZZY MORPHOLOGICAL OPERATORS (FMO)

An n-dimensional gray-scale image is model as an $\mathbb{R}^n \to [0,1]$ function. Taking two n-dimensional images A and B ("structuring element"), a conjunctor $\mathcal{C}$ and an implicator $\mathcal{I}$:

Definition 1 *[De Baets, 1997] The fuzzy dilation $D_{\mathcal{C}}(A,B)$ and fuzzy erosion $E_{\mathcal{I}}(A,B)$ of A by B are the gray-scale images defined by*

$$D_{\mathcal{C}}(A,B)(y) = \sup_x \mathcal{C}(B(x-y), A(x))$$

$$E_{\mathcal{I}}(A,B)(y) = \inf_x \mathcal{I}(B(x-y), A(x)).$$

Definition 2 *[De Baets, 1997] The fuzzy closing $C_{\mathcal{C},\mathcal{I}}(A,B)$ and fuzzy opening $O_{\mathcal{C},\mathcal{I}}(A,B)$ of A by B are the gray-scale images defined by*

$$C_{\mathcal{C},\mathcal{I}}(A,B)(y) = E_{\mathcal{I}}(D_{\mathcal{C}}(A,B), -B)(y)$$

$$O_{\mathcal{C},\mathcal{I}}(A,B)(y) = D_{\mathcal{C}}(E_{\mathcal{I}}(A,B), -B)(y).$$

In particular we choose a conjunctive (left-continuous) uninorm $\mathcal{U}$ and its residual implicator $I_{\mathcal{U}}$.

FMO: PROPERTIES

- Monotonicity, interaction with union and intersection,
- invariance under translating and scaling,
- inclusion properties, commutativity and associativity of fuzzy dilations,
- local knowledge, adjunction properties,
- combination properties of dilation and erosion,
- extensivity of dilation and fuzzy closing,
- anti-extensivity of erosion and fuzzy closing.
- Duality: we can use any representable conjunctive uninorm with its associted negation or any left-continuous idempotent uninorm defined by a negation.
- The opening and closing are idempotent: $C_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(C_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(A,B),B) = C_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(A,B)$, $O_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(O_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(A,B),B) = O_{\mathcal{U},\mathcal{I}_{\mathcal{U}}}(A,B)$.

In some properties we need that $B(0) = e$, where e is the neutral element of the considered uninorm.

CLOSE AND OPEN FUZZY OBJECTS

Definition 3 *Let A and B be two gray-scale images We say that A is B-closed (resp. B-open) if $C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(A,B) = A$ (resp. $O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(A,B) = A$).*

$C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(A,B)$ is B-closed, $O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(A,B)$ is B-open.

Properties:

- Characterization of a B-open (B-closed) fuzzy object as a dilation of a fuzzy object (erosion of a fuzzy object),
- extremal properties,
- preservation of B-openness and B-closedness by intersections and unions,
- duality between closed and open fuzzy objects

GENERALIZED IDEMPOTENCE

For representable uninorms the generalized idempotence of opening and closing can also be proved (see GRT04 for details):

Proposition 1 *Let $\mathcal{U}$ be a conjunctive representable uninorm. If A is B-open and rang(A) and rang(B) are finite sets, then for any fuzzy object F it holds:*

$$\begin{aligned} &O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,B),A) \\ &\qquad = O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,A),B) = O_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,A) \end{aligned}$$

and dually

$$\begin{aligned} &C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,B),A) \\ &\qquad = C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,A),B) = C_{\mathcal{U},\mathcal{I}_\mathcal{U}}(F,A) \end{aligned}$$

EXPERIMENTAL RESULTS

- Structuring elements:

Classical Umbra Approach	Fuzzy Approach
$\begin{pmatrix} 219 & 219 & 219 \\ 219 & 255 & 219 \\ 219 & 219 & 219 \end{pmatrix}$	$e \cdot \begin{pmatrix} 0.86 & 0.86 & 0.86 \\ 0.86 & 1.0 & 0.86 \\ 0.86 & 0.86 & 0.86 \end{pmatrix}$

- Fuzzy gradient:

$$D_{\mathcal{U}}(A,B) - E_{\mathcal{I}_{\mathcal{U}}}(A,B) \Longrightarrow \text{Edge detection}$$

- Input images used in the experiments:

EXPERIMENTAL RESULTS

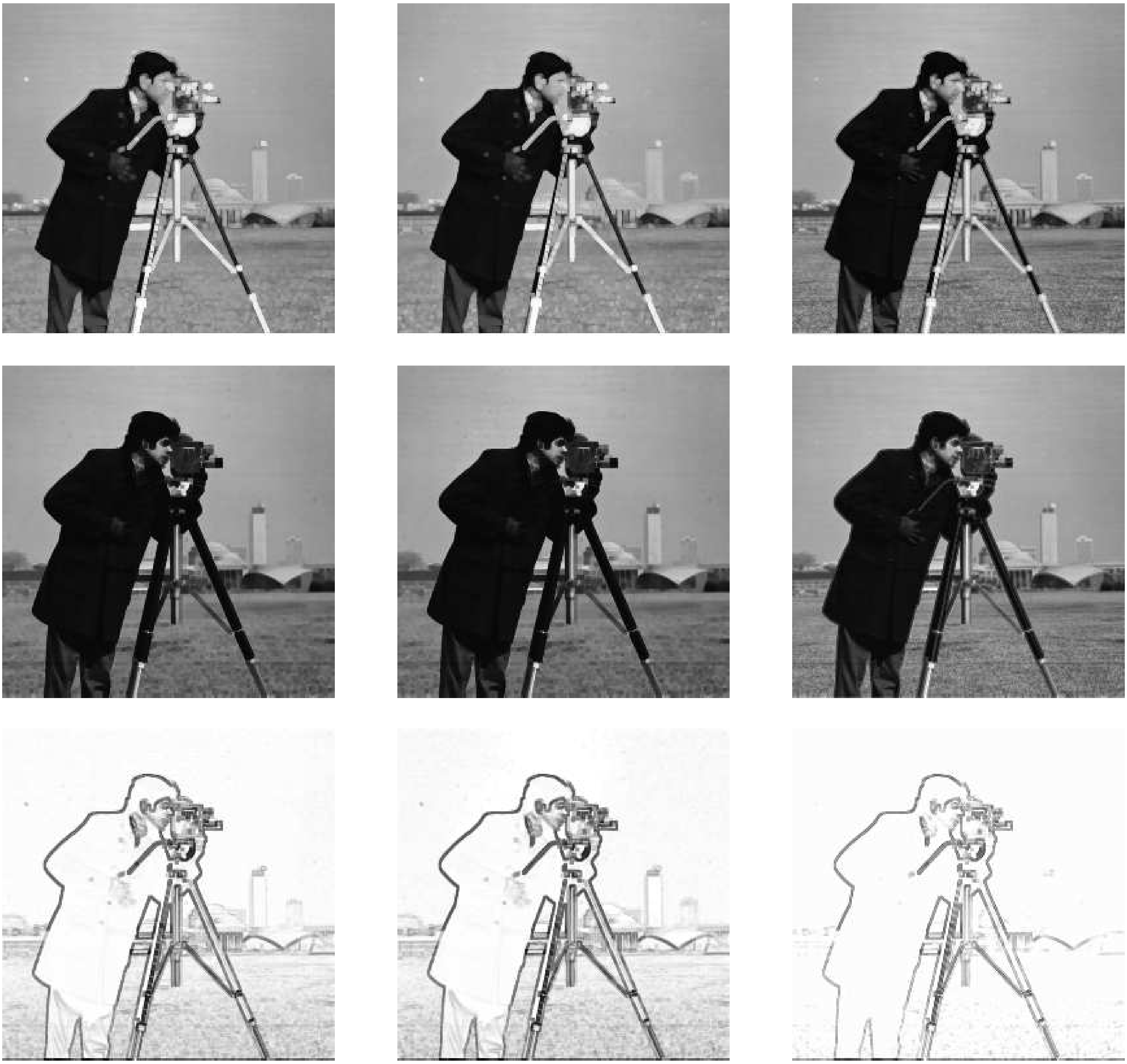

Figure 1: From top to bottom, dilation, erosion and gradient obtained using **idempotent uninorms** with negation $\mathcal{N}(x) = \sqrt{1-x^2}$ and $\mathcal{N}(x) = 1-x$, respectively. Right: **umbra approach**

EXPERIMENTAL RESULTS

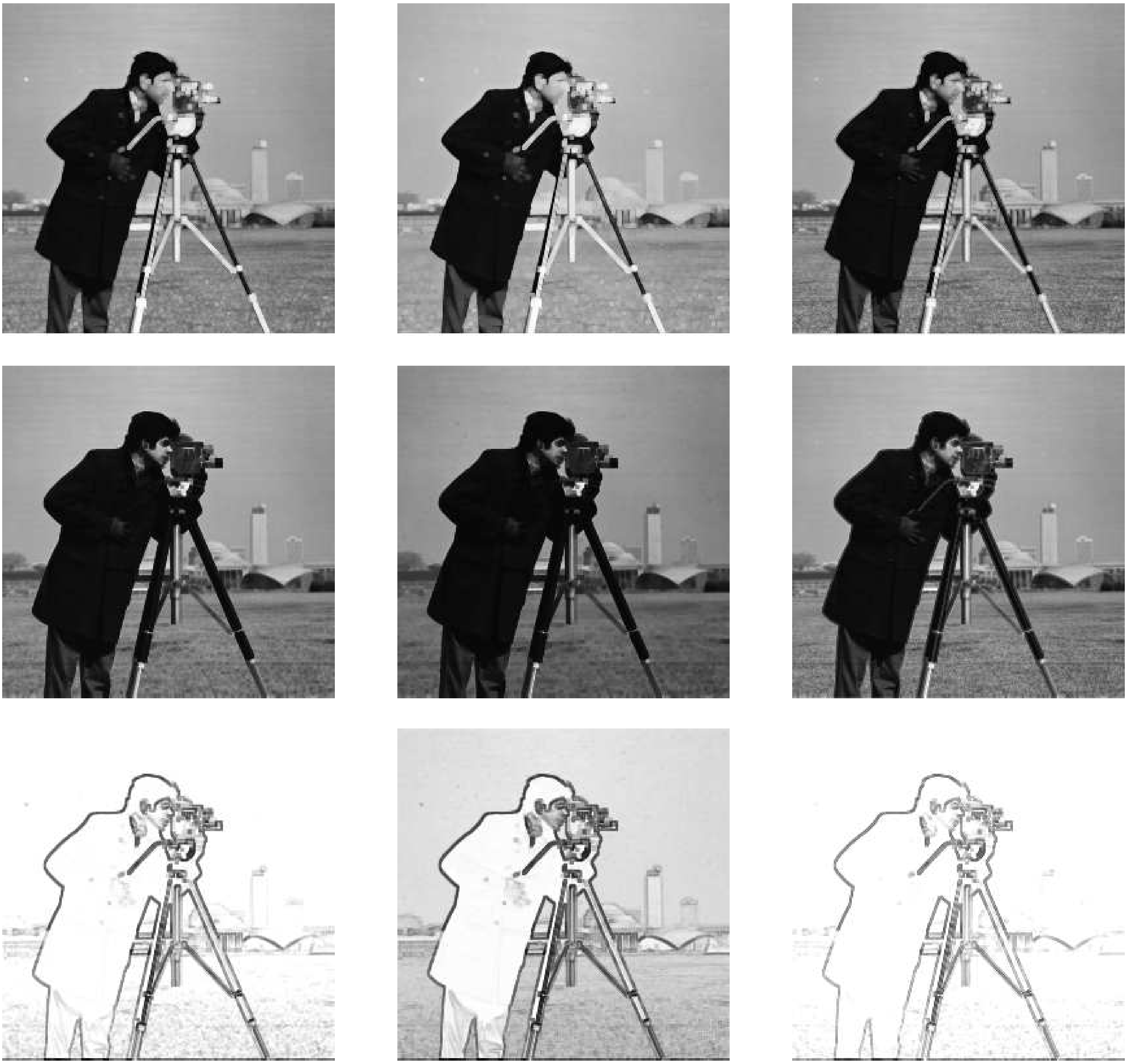

Figure 2: Fuzzy dilation, fuzzy erosion and fuzzy gradient obtained using **representable conjunctive uninorms** with additive generator $h(x) = \ln(x/(1-x))$, $h(x) = (x-0.5)/(x(1-x))$, and the **Lukasiewicz** t-norm.

EXPERIMENTAL RESULTS

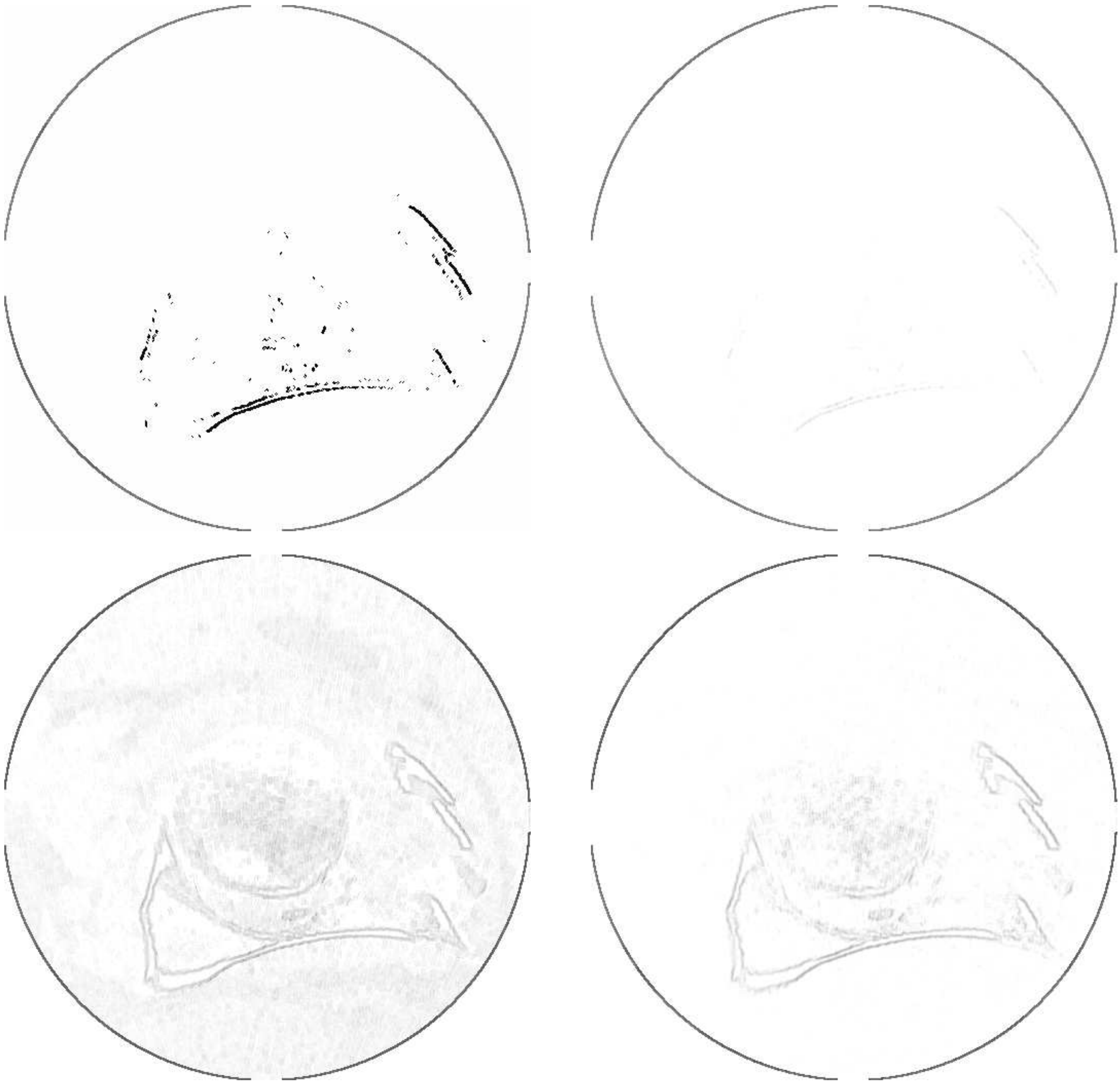

Figure 3: Several gradient images. Top: left, umbra approach; right, Lukasiewicz t-norm. Bottom: left, idempotent uninorm with negation $\mathcal{N}(x) = \sqrt{1-x^2}$; right, representable conjunctive uninorm with additive generator $h(x) = \ln(x/(1-x))$.

EXPERIMENTAL RESULTS

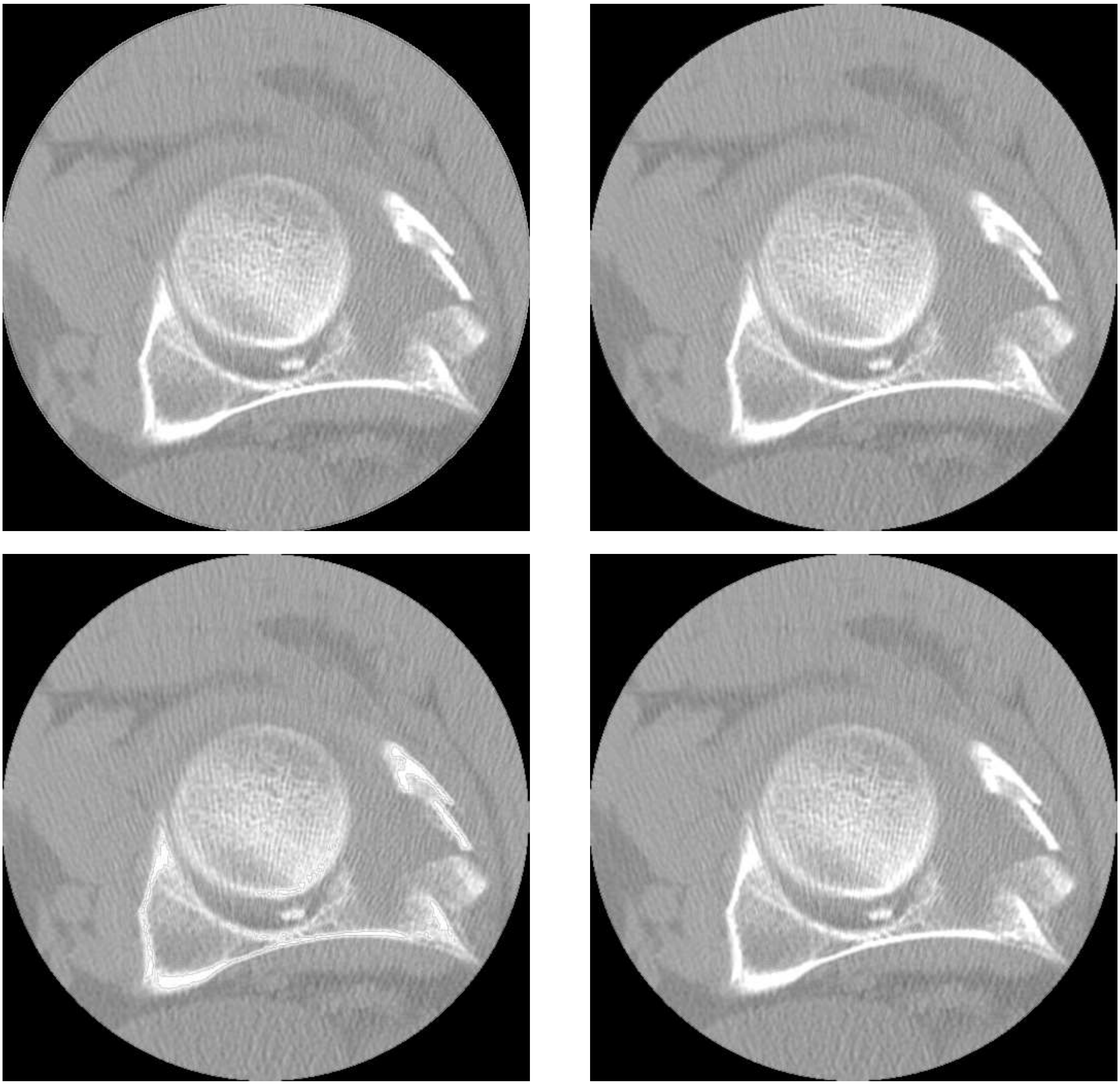

Figure 4: Top to bottom: closing and opening. Left to right: umbra approach, fuzzy operator using **Lukasiewicz** t-norm.

EXPERIMENTAL RESULTS

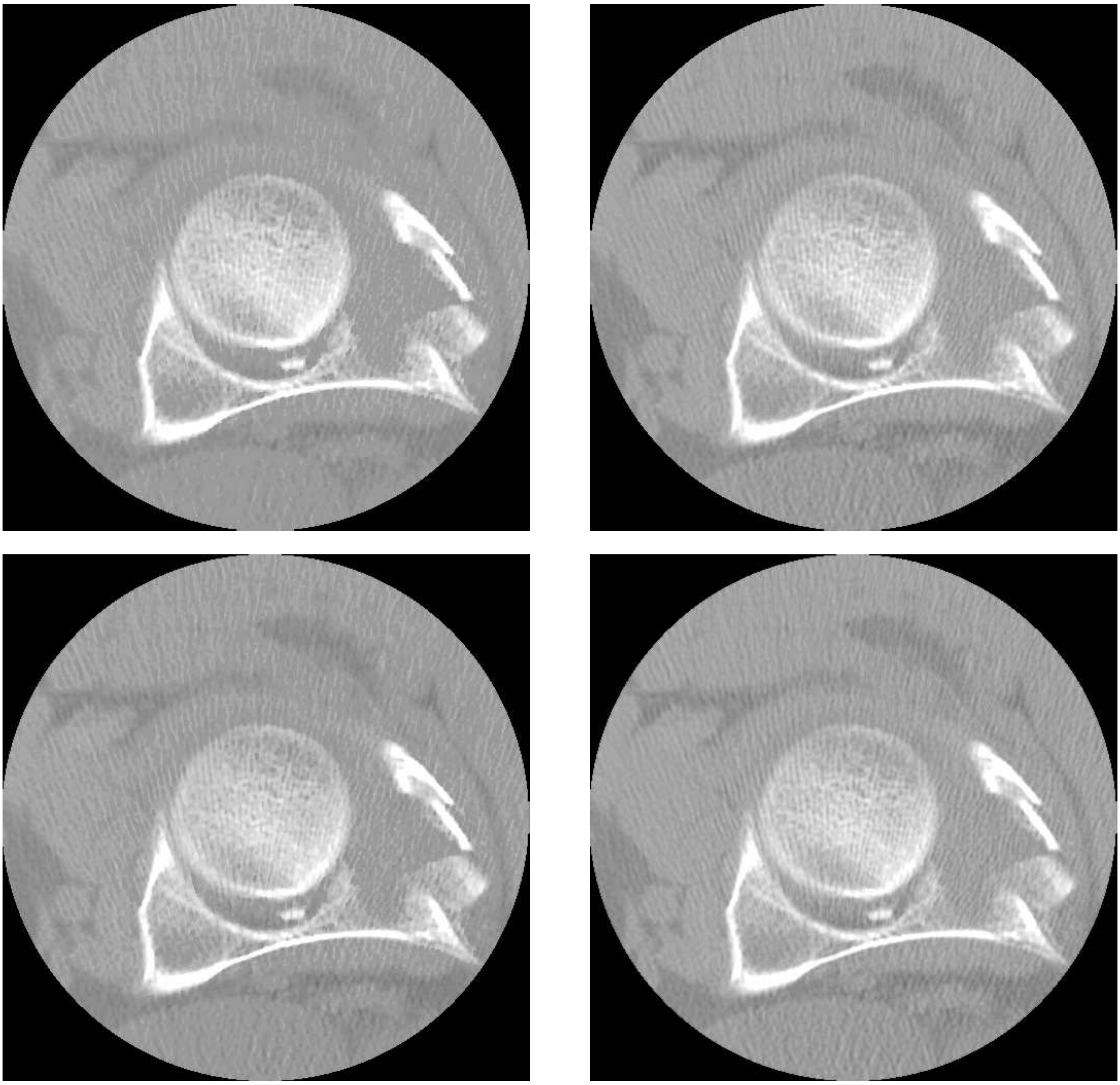

Figure 5: Top to bottom: closing and opening. Left to right: fuzzy operators using idempotent uninorm with negation $\mathcal{N}(x) = \sqrt{1-x^2}$, and fuzzy operators using representable conjunctive uninorm with additive generator $h(x) = \ln(x/(1-x))$.