Color images filtering by anisotropic diffusion

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Abstract: In this paper we propose a model of partial differential equations (PDE's) for color images diffusion. The model is based on the hypothesis that the essential 'geometry' of natural color images is contained in the set of level lines of the luminance component. The proposed PDE's perform an anisotropic diffusion of the color components in the orthogonal direction to the gradient of the luminance, thus preserving the edges of the image. Existence and uniqueness of solutions for the model are stated and a numerical scheme for its practical implementation is proposed. Finally, some experiments, including some applications to noise removal, are shown.

Keywords: color images filtering; PDEs-based anisotropic model.

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1 INTRODUCTION

Most diffusion models in image processing smooth the images by modifying them via a partial differential equation or PDE. In the isotropic linear filtering, the model is given by the heat equation $\frac{\partial I}{\partial t} = \triangle I$, where I is the filtered image, $\triangle$ is the Laplacian operator, and the original (grey-scale) image I_0 is used as the initial condition. It is known that to solve the heat equation is equivalent to convolve the initial image with a Gaussian kernel G_σ, for some σ. As a result of this smoothing the edges of the images become blurred (see Figure 1).

An attempt to filter the images while trying to preserve their edges is given in the work of Perona and Malik [5]. They introduce the idea of anisotropic diffusion, that is, to smooth the image in the orthogonal direction to the gradient, $DI^\perp$, and prevent, as much as possible, the diffusion across the direction of the edges (i.e. the direction of the gradient, DI). They replaced the heat equation by the following one

$$\frac{\partial I}{\partial t} = div(g(|DI|)DI),$$

where $DI = (I_x, I_y)$ (subscripts denote derivatives) and $g(|DI|)$ is an edge-stopping function (diffusion is "stopped" across edges).

In [1], Alvarez et al. propose the mean curvature equation as an anisotropic diffusion model. The associated PDE can be written as

$$\frac{\partial I}{\partial t} = D^2 I\left(\frac{DI^\perp}{|DI|}, \frac{DI^\perp}{|DI|}\right) \quad (1)$$

where the operator $D^2 I(\frac{DI^\perp}{|DI|}, \frac{DI^\perp}{|DI|})$ denotes the second derivative of I in the orthogonal direction to the gradient.

This PDE is very close to the Perona and Malik model: diffusion occurs only along the direction $DI^\perp$ and it is "stopped" in the direction of the gradient. After these pioneer works, anisotropic diffusion models for filtering and enhancement of images have been studied by several authors (see for instance [4] for a review on the subject). Figure 1 shows an example of the difference between isotropic and anisotropic diffusion.

The previous approaches have focused on the filtering of grey-scale images. In the recent years, a number of authors have addressed the problem of color image diffusion and enhancement by using PDE's models for multi-valued images. The simplest way to handle these images is to smooth each color component (or 'channel') independently of the others. This is equivalent to consider each channel as an independent grey-scale image, with its own edges, which is filtered using some PDE. The filtered color im-

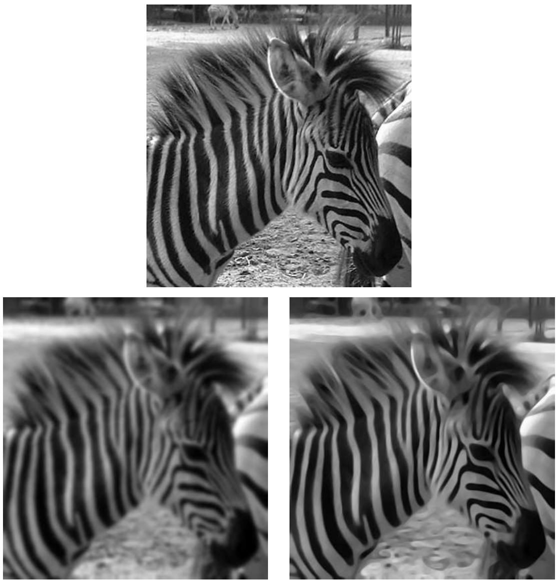

Figure 1: Top row, original image. Bottom row, isotropic (heat equation, left) versus anisotropic (mean curvature, right) diffusion.

age is then obtained by combining together the filtered channels. The notion of 'channel' depends on the method used to represent color information. For instance, it can be 'red', 'green', and 'blue' when using the RGB color system, or 'hue', 'saturation' and 'illumination' when using HSI.

Some attempts have been made to process together the color information when filtering color images. In a preliminary work [3], A. Chambolle proposes to extend the theory of multiscale analysis of images introduced in [1] to color image processing and he discusses some examples of PDE's. In [7], the authors propose an evolution equation for anisotropic diffusion of multivalued images where a first approximation to the gradient for this kind of images is given. Another approach for multivalued image filtering is given by B. Tang et al. in [8], where an algorithm that separates the color data into chromaticity and brightness is proposed and then each one of the components is filtered by a diffusion equation adapted from the theory of p-harmonic maps.

Following these approaches that consider the color image as a whole, we propose a filtering method that tries to preserve the 'essential geometry' of the color image. In [2], Caselles et al. give a model based on the hypothesis that the essential shape information (or 'geometry') of a natural color image is contained in the set of the level lines of the illumination channel, the so called **topographic map**. The hypothesis is tested using an experimental procedure where the chromatic components, saturation and hue, of several images are forced to have the same topographic map than the luminance component. In the case of natural, non-synthetic, images, the modified images turn out to be visually identical to the original ones, which seems to support the proposed hypothesis.

Based on this hypothesis, we give a new model for color anisotropic diffusion. This model can be interpreted as a diffusion of the color components in the orthogonal direction to the gradient of the luminance. That is, when each color component is filtered, the goal is not to preserve its own edges, but the 'edges' (i.e. the topographic map) of the luminance component. Therefore, we propose the following coupled PDE's system for color image diffusion

$$\frac{\partial u_i}{\partial t} = D^2 u_i \left(\frac{DL^\perp}{|DL|}, \frac{DL^\perp}{|DL|} \right) \quad i = 1, 2, 3 \qquad (2)$$

where $\mathbf{u} : \mathbb{R}^2 \to \mathbb{R}^3$ is the vector-valued image, $\mathbf{u} = (u_1, u_2, u_3)$, and $L : \mathbb{R}^2 \to \mathbb{R}$ is the luminance component.

In Section 2 , we propose an equivalent mathematical well-posed model of system (2) and we give an analytical result about the existence and uniqueness of solution. Some aspects of the numerical scheme used for the computations are commented in the same section. Section 3 displays some experiments from which some conclusions are inferred in Section 4 .

2 THE MODEL

In order to support theoretically the aforementioned model, we propose an equivalent one which is mathematically well-posed.

Suppose that we have defined a bounded vector field, that is, $W : \mathbb{R}^2 \times [0, \infty) \to \mathbb{R}^2$ and let $f : [0, \infty) \to [0, 1]$ be a smooth function defined as

$$\begin{cases} f(s) = 1 & \text{if } s < \epsilon \\ f(s) = 0 & \text{if } s > 2\epsilon. \end{cases}$$

Then, we propose the following model for the filtering of each color channel $u = u_i$

$$\begin{aligned} u_t = \quad & f(|W(x,t)|) D^2 u \left(W^\perp(x,t), W(x,t)^\perp \right) + \\ & + D^2 u \left(W(x,t), W(x,t) \right), \end{aligned} \qquad (3)$$

where W is the orthogonal direction to the gradient of L, the luminance of the initial image. We have the following result for system (3):

Theorem 1. *Let $W(x,t)$ be Lipschitz continuous in the space variable, and bounded. Then, for a given continuous initial condition u_0, equation (3) has a unique viscosity solution.*

Remarks.

• The system of PDE's (3) performs the diffusion of the color components in the direction orthogonal to the gradient of the luminance and defines a kind of anisotropic multi-scale representation of the initial image $\mathbf{u}$. Since we have made the hypothesis that the geometry of a natural image is contained on the level lines of its luminance, these PDE's permit the filtering of the image while keeping its geometry.

• W can also be defined as the orthogonal direction to the gradient of a smoothed version of L, obtained by applying equation (1) to the luminance component of the original image.

2.1 Numerical scheme

We construct a convergent finite difference scheme for system (3) which is monotone and consistent. The scheme is implemented on a uniform rectilinear grid and the basic idea is to search for the direction ξ which better approximates W in every point of the grid. Then, we apply a finite differences scheme to the equation

$$u_t = f(|W|)u_{\eta\eta} + u_{\xi\xi},$$

where η denotes the orthogonal direction to ξ .

Concerning the choice of a color system to represent the color images, several options may be considered. The most common ones are RGB and HSI. In the RGB case, the luminance function L is computed as the mean value of the three components. In the HSI case $L \equiv I$ and for the hue component H (an angular value) we approximate its derivatives using the approach proposed by Perona in [6] for the diffusion of oriented values.

3 EXPERIMENTAL RESULTS

We display some examples of the application of the proposed model. In all the experiments colors are represented in RGB color space. Each component is then diffused according to equation (3) following the direction orthogonal to the gradient of the luminance of the original (non-filtered) image. The images in this section can be found at: http://dmi.uib.es/~lisani/colorpde/imagesweb.pdf.

Figure 2 displays the original and filtered images and Figure 3 shows a detail of the image filtered with an increasing number of iterations.

In the second example the model has been used to filter out the characteristic noise of a digital picture obtained with a CCD camera. Figure 4 displays the original (top) and the filtered images (bottom), while Figures 5 and 6 show two details of the filtering. Observe how noise is effectively removed while edges are kept.

4 CONCLUDING REMARKS

Our aim has been to propose a new diffusion model that explicitly agrees with the hypothesis that the geometry of natural color images is contained in the luminance component. The experimental results seem to validate the proposed diffusion model, in the sense that the geometry of the natural images (that is, the shapes of the objects in the scene) is preserved while the rest is smoothed. Indeed, the model can be used to remove noise, as shown in the previous experiments.

Figure 2: Top row, original image. Bottom row, anisotropic diffusion of the image after 10 iterations of the scheme.

Figure 3: Top row, a detail of the original image. Bottom row, anisotropic diffusion of the image. Left, result of diffusion after 5 iterations. Right, result after 10 iterations.

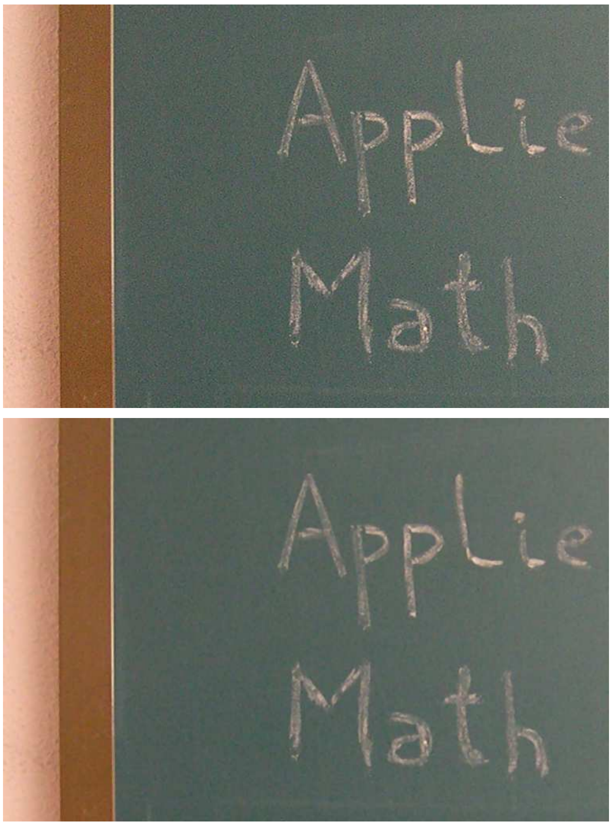

Figure 4: Top row, original image. Bottom row, anisotropic diffusion of the image after 10 iterations of the scheme.

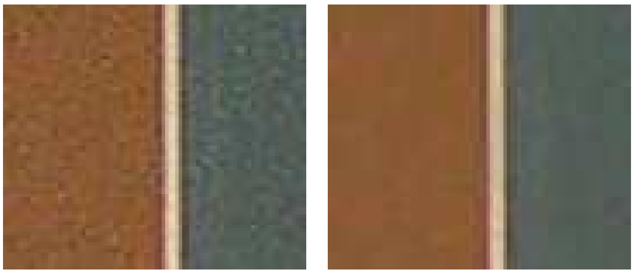

Figure 5: Left, a detail of the original image. Right, anisotropic diffusion after 10 iterations.

Figure 6: Left, a detail of the original image. Right, anisotropic diffusion after 10 iterations.

The model is well-posed and we have constructed a numerical scheme which is convergent to the solution .

In our future research we plan to apply model (3) using a general vector field W. Several options can be considered: the orthogonal direction to the gradient of the less noisy channel or of any linear combination of the three channels, etc.

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