Improving the reconfigurable SOVA/log-MAP turbo decoder for 3GPP

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Abstract: SOVA and log-MAP turbo decoding algorithms are the two prime candidates for decoding turbo codes. They share common operations, making feasible a reconfigurable SOVA/log-MAP turbo decoder for reduced power consumption. Using a common scaling factor in the extrinsic information calculation of both algorithms can improve performance with minimal effort. The scaling factor is independent of the signal to noise ratio. Simulations with the 3GPP parameters over AWGN and uncorrelated Rayleigh fading channels, show improvements over standard algorithms of up to 0.22 dB and 0.5 dB for log-MAP and SOVA respectively.

1. INTRODUCTION

Turbo coding [1] has been adopted as a channel coding scheme for several 3rd generation mobile systems, in particular 3GPP (third generation partnership project) for high data rates. The soft-input/soft-output (SISO) decoder is the critical part of the decoder, using the soft output Viterbi algorithm (SOVA) [2], [7] or the log maximum a posteriori algorithm (log-MAP) [3], [7]. Log-MAP gives better performance than SOVA, but SOVA is less complex [3]: the algorithms share some operations [4]. Since for different applications (e.g. video, data transfer) and for different parameters (e.g. performance, complexity) either SOVA or log-MAP is optimum, a reconfigurable SOVA/log-MAP turbo decoder can be used, resulting in lower power consumption [10].

The reason for considering only SOVA and log-MAP algorithms for a reconfigurable turbo decoder is explained by the following. In 3GPP standard for real time applications we want the lowest possible latency, while BER (bit error rate) is not a priority. On the other hand, for non-real time applications we want the lowest possible BER, while latency is not a priority [10]. The MAP algorithm is not considered because it has high complexity and suffers from numerical problems in practice and because MAP gives almost the same performance as log-MAP [3].

For an encoder memory M=3 the number of operations [3] using log-MAP is: 38+129+8+38=213. For SOVA the number of operations is: 20+24+8+24=76, while for max-log-MAP: 38+91+8=137. It is obvious that log-MAP is 2.8 times more complex than SOVA while max-log-MAP is 1.8 times more complex than SOVA. Thus, although from a latency point of view SOVA is the best of the three turbo decoding algorithms, from a performance point of view log-MAP is the best.

2. RECONFIGURABLE DECODER

In [4] the reconfigurable SOVA/log-MAP algorithm is analysed: here, only basic operations are described. The branch metric calculation is common for SOVA and log-MAP. For a code with rate 1/3

\[
D(s_i, s_{i+1}) = x_i^s (z_i + y_i^s) + x_i^{p1} \cdot y_i^{p1} + x_i^{p2} \cdot y_i^{p2}
\]

(1)

where \(x^s\) and \(x^{p1}\), \(x^{p2}\) are the systematic and the parity outputs of the turbo encoder respectively for the state transition \(s_i\) to \(s_{i+1}\). The symbols \(y^s\) and \(y^{p1}\), \(y^{p2}\) are received symbols, and \(z_i\) is a priori information for symbol index \(i\) from the previous decoder. Applying the Jacobian logarithm to MAP results in log-MAP [3], [7], [9]:

\[
\max^a(m, n) = \ln\left[ e^m + e^n \right] = \max(m, n) + \ln\left( 1 + e^{-|m-n|} \right)
\]

(2)

Here, \(m\) and \(n\) are real numbers. Equation (2) shows the commonality between SOVA and log-MAP [4]. For log-MAP the forward state metric \(a(s_i)\) for state \(s_i\) and symbol \(i\) is calculated as follows:

\[
a(s_i) = \max_{s_{i-1} \in A} \left( a(s_{i-1}) + D(s_{i-1}, s_i) \right)
\]

(3)

where \(A\) is the set of states \(s_{i-1}\) connected to state \(s_i\). For SOVA the (forward) state metric \(\Gamma(s_i)\) is:
\[
\Gamma(s_i) = \max_{s_{i-1} \in A} \left( \Gamma(s_{i-1}) + D(s_{i-1}, s_i) \right)
\] (4)

The difference \(\Delta(s_i)\) between the two state metrics that are compared is:

\[
\Delta(s_i) = \max_{s_{i-1} \in A} \left( \Delta(s_{i-1}) + D(s_{i-1}, s_i) \right) - \min_{s_{i-1} \in A} \left( \Delta(s_{i-1}) + D(s_{i-1}, s_i) \right)
\] (5)

For log-MAP the backward state metric \(b(s_i)\) is:

\[
b(s_i) = \max_{s_{i+1} \in B} \left( b(s_{i+1}) + D(s_i, s_{i+1}) \right)
\] (6)

where \(B\) is the set of states \(s_{i+1}\) connected to state \(s_i\). Equations (3) and (6) also have common operations [4]. The SISO decoder soft output \(\Lambda_i\) for symbol \(i\) and for log-MAP is:

\[
\Lambda_i = \max_{s_1} \left( a(s_1) + D(s_1, s_{i+1}) + b(s_{i+1}) \right)
\] (7)

where \(S_1\) and \(S_0\) are the sets of all state transitions associated with a bit 1 and 0 respectively. For SOVA, \(\Lambda_i\) is calculated according to [2], [7]:

\[
\Lambda_i = (2m_i - 1) \rho_i
\] (8)

where \(m_i\) is the estimated bit sequence computed by tracing \(\Gamma(s_i)\) back in the trellis, and \(\rho_i\) is a reliability sequence computed by using \(\Delta(s_i)\) for every symbol \(i\) (see [2]).

Using \(\Lambda_i\), the extrinsic information \(L_{ex_i}\) is calculated. After interleaving, this provides a priori information in the other SISO decoder:

\[
L_{ex_i} = \Lambda_i - \frac{4 \cdot \alpha_i \cdot E_s}{N_0} y_i^s - z_i
\] (9)

with \(\alpha_i\) the fading amplitude for symbol \(i\) and \(E_s/N_0\) the signal to noise ratio (SNR).

### 3. SCALING THE TURBO DECODER

In [5] it is noted that SOVA suffers from two distortions: overoptimistic soft outputs, and correlation between the intrinsic and extrinsic information. Performance is degraded substantially by the first type of distortion but only mildly by the second. Therefore, only the first type of distortion, which depends on the SNR, is considered. The compensation coefficient must be calculated for each decoder stage and recalculated for each new frame. For an AWGN channel the slight increase in computational complexity is justified by a significant performance improvement between 0.2 and 0.4 dB [5].

In this work we show that for a reconfigurable SOVA/log-MAP decoder compensation of the extrinsic information is possible with a common scaling factor which is constant and independent of the SNR. Particularly for SOVA we show that the same or even better improvement can be achieved as in [5] with the scaling factor independent of SNR and constant over all iterations and turbo interleaver lengths.

As in [6] for max-log-MAP decoding algorithm, we apply a scaling factor \(s\) to the calculation of \(L_{ex_i}\):

\[
L_{ex_i} = \left[ \frac{\Lambda_i}{2} - \frac{4 \cdot \alpha_i \cdot E_s}{N_0} y_i^s - z_i \right] s
\] (10)

### 4. SYSTEM MODEL

The system model that is used for the simulations is shown in Figure 1. The 3GPP standard parameters are used [8]. The information bits (or symbols) \(u_i\) are grouped into frames whose length must be \(\geq 40\) and \(\leq 5114\). The output bits of the turbo encoder are then modulated using a Binary Phase Shift Keying (BPSK) modulator. The output of the BPSK modulator \(y_k\) is multiplied by fading amplitudes \(a_k\) and noise \(n_k\) is added to produce the received value \(r_k\). This value is not quantized, therefore floating point arithmetic is used. Then \(r_k\) is BPSK demodulated and turbo decoded. The output of the turbo decoder is an estimate \(u_i^\prime\) of the information bit \(u_i\).

![Figure 1: System model](image)

### 4.1 Encoder

The turbo encoder [1], [7], [9] is made up of two \(\frac{1}{2}\) rate Recursive Systematic Convolutional (RSC) encoders, each with constraint length \(K=4\) and octal polynomials 13 (feedback) and 15 (redundancy). The two encoders are concatenated in parallel and separated by an interleaver with parameters as
specified in [8]. The rate of the encoder is 1/3. Furthermore, the two RSC encoders of the turbo encoder are left open (no tail bits used).

4.2 Channel

The encoded bits are transmitted as shown in Figure 1. With appropriate sampling, the discrete representation of this channel is \( r_k = a_k y_k + n_k \) where \( k \) is an integer bit index (here \( k=3i \): every symbol has 3 bits), \( y_k \) is a BPSK symbol amplitude and \( n_k \) is a AWGN component with zero mean and power spectral density \( N_0/2 \). The variable \( a_k \) is a fading amplitude that may vary from code bit to code bit. When the AWGN channel model is used \( a_k = 1 \) for all code bits. For the uncorrelated Rayleigh fading channel model \( a_k \) is generated according to the following formula for each bit \( k \) [9]:

\[
a_k = \sqrt{b_k^2 + c_k^2}
\]

(11)

where \( b_k \) and \( c_k \) are zero mean statistically independent Gaussian random variables each having a variance \( \sigma^2 = 0.5 \).

4.3 Decoder

A simplified diagram of the turbo decoder [1], [7], [9] showing the compensation operation, without details of interleaving and deinterleaving, is presented in Figure 2. Turbo decoding is performed iteratively, with each SISO decoder using information from the previous step. Eight iterations are used in the decoder and no quantisation is used at the reception of the data. The symbol \( s \) denotes the scaling factor.

5. SIMULATION RESULTS

The proposed turbo coded system is simulated using AWGN and uncorrelated Rayleigh fading channels. Two example frame lengths are used to show the effect of the scaling factor on the performance: a short length of 100 bits and the maximum allowed value for the 3GPP standard (5114 bits).

Figure 3 shows the performance of both algorithms for the scaling factor giving best results \((s=0.7)\) compared to the standard algorithms \((s=1)\) for a frame length of 100 bits and AWGN channel. An improvement of 0.1 dB for log-MAP and 0.35 dB for SOVA is seen at a BER of \( 6 \times 10^{-3} \).

Figure 4 shows that for a frame length of 5114 bits and AWGN there is an improvement of 0.2 dB for log-MAP at a BER of \( 6 \times 10^{-3} \). For SOVA there is an improvement of 0.35 dB at the same BER.

Figures 5 and 6 show the performance in an uncorrelated Rayleigh fading channel. For the short interleaver of 100 bits there is an improvement of 0.18 dB for log-MAP and 0.4 dB for SOVA at a BER of \( 6 \times 10^{-3} \). In figure 5 and for 4 dB it can be seen that there is no improvement for log-MAP. This is because of the small interleaver length. For 5114 bit frame length and at a BER of \( 6 \times 10^{-3} \) an improvement of 0.22 dB is found for log-MAP, while for SOVA an improvement of 0.5 dB is possible (figure 6).

6. CONCLUSIONS

In this work it is shown that it is possible to improve the performance of a reconfigurable SOVA/log-MAP turbo decoder by applying a simple common scaling factor to the extrinsic information. Using the 3GPP standard specifications the performance is improved by some 0.35 to 0.5 dB for SOVA and 0.1 to 0.22 dB for log-MAP at a BER of \( 6 \times 10^{-3} \). For SOVA the same (for AWGN channel) or even better (for uncorrelated Rayleigh fading channel) improvement can be achieved as in [5] with the scaling factor independent of SNR. The scaling factor is constant over all frame lengths and iterations, and so can be implemented in a reconfigurable SOVA/log-MAP turbo decoder for 3GPP with minimum cost.

7. REFERENCES


Figure 3: Performance of 3GPP turbo code using a reconfigurable SOVA/log-MAP decoder with different scaling factors, frame length 100 bits, AWGN channel and 8 iterations

Figure 4: Performance of 3GPP turbo code using a reconfigurable SOVA/log-MAP decoder with different scaling factors, frame length 5114 bits, AWGN channel and 8 iterations

Figure 5: Performance of 3GPP turbo code using a reconfigurable SOVA/log-MAP decoder with different scaling factors, frame length 100 bits, uncorrelated Rayleigh fading channel and 8 iterations

Figure 6: Performance of 3GPP turbo code using a reconfigurable SOVA/log-MAP decoder with different scaling factors, frame length 5114 bits, uncorrelated Rayleigh fading channel and 8 iterations